

MODULE SEVEN

This module addresses systems of linear equations.

SC Academic Elementary Algebra Standards included in this module are:

- EA-5.11 Analyze given information to write a system of linear equations that models a give problem situation.
- EA-4.9 Carry out a procedure to solve systems of two linear equations graphically.
- EA-4.10 Carry out a procedure to solve systems of two linear equations algebraically.
- EA-5.8 Analyze the equations of two lines to determine whether the lines are perpendicular or parallel.

Who Wins the Race?

Goal: The goal of this activity is to introduce systems of linear equations to students using a real world connection prior to starting Lesson #1. This inquiry-based approach allows students to discover the general patterns and relationships between two linear functions using a variety of representations without labeling it as a linear system.

Standards: The extensions of this initial activity address the following Algebra I standards:

- EA-4.1 Carry out a procedure to write an equation of a line with a given slope and y-intercept
- EA-4.9 Carry out a procedure to solve systems of two linear equations graphically.
- EA-4.10 Carry out a procedure to solve systems of two linear equation algebraically.
- EA-5.7 Apply the concept of slope as a rate of change to solve problems
- EA-5.11 Analyze given information to write a system of linear equations that models a given problem situation.

Teacher Notes:

Procedure:

1. Students should be groups of 4, if possible. Each group is given one of the three scenarios. Assign the following roles: recorder, reader, and calculator and a checker. More than one group will have the same scenario based on the size of your class.
2. Each group is to complete the table of values, the graph and answer the questions for their scenario.
3. Once each group is finished, the checker will display their answer on the board. Have groups with the same scenario display their answers in the same area of the board in order to compare solutions.

Connections:

*The **overarching theme** of this problem is to have students explore mathematical concepts without formally naming those concepts. The students are asked simply to solve the problem and answer some questions. Sample introduction: “I have three different problems that I would like for you to solve. In order to solve these problems, you will use three concepts that we have previously discussed. They are creating a table of values, graphing points and rate of change.” Based on the level of your students, you may need to review using the constant rate of change to build a table of values.*

*With the **teacher as facilitator**, students are building on their prior knowledge of the table of values, constant rate of change and graphing to develop mathematical models for linear systems.*

Once these models are developed, the teacher can make formal connections by asking additional questions.

As you ask the discussion questions below, it is important that students are recording their thoughts. You may want to have the students write the question and their answers or simply have them summarize the big ideas.

4. **Are there other ways that two lines can interact with each other?** This establishes the idea that there are three models for a system of two linear equations. Have the students describe each scenario in their own words. Sample answers are they cross, they don't cross and they are on top of each other.
5. **What do you notice about the rate of change in each problem? What about the starting point for each runner? How do those two factors affect the graph?** Go through each scenario and address each question. This leads to the idea that parallel lines have the same rate of change and different y-intercepts. Lines that coincide have the same rate of change and same y-intercept and intersecting lines have different rates of change and different y-intercepts.
6. **So in mathematical terms what do these three scenarios represent?** At this point you may want to formally define the term system and then the term system of linear equations. You may also want to formally name each model. For example, exactly one solution, no solution or infinitely many solutions (as per the SC Algebra I support document)
7. **Explore the concept of a solution.** Share with them that when they are asked to solve a system, they are basically being asked to describe how the two graphs are interacting with each other; re-emphasizing the formal label from #6. Next, determine the solution for each scenario.
8. **Multiple representations:** For each scenario, it is important to reinforce the concept of a solution using each representation (graphic, tabular and algebraic). Discuss how to determine where the solution is on the graph and in the table. For example, ask "how does this table of values indicate that the system has exactly one solution? Where is it on the graph?" Continue this for each scenario.

At this point, you may want the students to reflect on their understanding of systems thus far. Based on the length of your class period this may be a place to stop.

9. **Representing systems algebraically.** Remind students that we have looked at the graphic and tabular representations of a linear system. At this point in the course, students should be able to write the equations for each runner. This is an opportunity to review slope intercept form, $y = mx + b$ where m is the slope (how much you move) and b is the y-intercept (where you begin). Once the equations are written, you can use the parametric mode in your calculator to model the motion of the two runners. A calculator how-to sheet is attached. Kids like it!!

At this point, you can discuss that there are a variety of methods that can be used to solve a linear system. One method requires the use of the graph and the others require the use of the equations. First, we will explore the graphical approach.

Who Wins The Race?

Scenario One



Melissa and Takeya are practicing for the 40 yd dash. Melissa challenges Takeya to a race. Confident in her abilities, Melissa tells Takeya to go 10 yards ahead before they begin the race. Melissa is running at a rate of 5 yd/sec and Takeya is running at a rate of 2.5 yd/sec. Will Melissa catch Takeya? If so, at what time and distance? Who will win? Justify your answers using the table of values and the graph.

Complete the table of values

Time (seconds)	Melissa's Distance (yd)	Takeya's Distance (yd)
0		
1		
2		
3		
4		
5		
6		
7		

Graph the data from your table. Name the axes and label each axes with an appropriate scale.

Who Wins The Race?

Scenario Two



Yang and Carrie are practicing for the 40 yd dash. Both Yang and Carrie start at the same time and are both running at a rate of 5 yd/sec. If they keep up their pace, who will finish first? Justify your answer using a table of values and the graph.

Complete the table of values.

Time (seconds)	Yang's Distance (yd)	Carrie's Distance (yd)
0		
1		
2		
3		
4		
5		
6		
7		
8		

Graph the data from your table. Label the axis. Label each axes with an appropriate scale

Who Wins The Race?

Scenario Three



John and Hector are running after school to get ready for the 40 yd dash. John challenges Hector to a race. They agree that the race will begin when Hector is 10 yards ahead. Both Hector and John are running at a rate of 5 yd/sec. Will John catch up with Hector? If so, at what time and distance? Who will win? Justify your answer using the table of values and the graph.

Complete the table of values.

Time (seconds)	John's Distance (yd)	Hector's Distance (yd)
0		
1		
2		
3		
4		
5		
6		
7		

Graph the data from your table. Name the axes and label each axes with an appropriate scale

Lesson # 1
Topic: Writing systems of linear equations
Standard (s): EA – 5.11

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In eighth grade, students generate and solve complex abstract problems that involve modeling physical, social, or mathematical phenomena (8-1.1).
 - In Elementary Algebra students will write a system of linear equations that models a given problem situation.
 - In Intermediate Algebra, students analyze a problem situation to determine a system of linear inequalities that models the problem situation (IA-2.3)
- **Taxonomy**
 - 4.3 B
 - Cognitive Process Dimension: Analyze
 - Knowledge Dimension: Conceptual Knowledge
- **Key Concepts**
 - System of linear equations
 - Modeling

II. Teaching the Lesson

In this lesson, students write systems of linear equations given a problem situation. This lesson precedes solving linear systems because writing linear systems provides a context and purpose for solving linear systems. Students have the pre-requisite skills needed to write each linear equation in the

system. Allowing students to brainstorm how to determine the solution using other methods provide a purpose for utilize more formal algebraic methods such as linear combination (elimination) and substitution.

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Assign variables to quantities for a given problem situation.
- Express relationships between variables representing quantities in a given problem situation.
- Write a system of linear equations given two linear relationships between two variables.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- I have thirteen coins consisting of quarters and dimes. The sum of money I have is \$2.20. Write a system of equations that could be solved to determine how many of each coin that I have.
- You are considering a membership in two health clubs. One has a \$200 down payment and monthly payments of \$15. The other has a \$30 down payment and monthly payments of \$50 dollars. Write a system of equations that could be solved to determine how many months you have to be a member before plan one becomes a better deal?

- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Solve the system of equations
- Set up equations that are not 2×2 linear systems.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- I have thirteen coins consisting of quarters and dimes. The sum of money I have is \$2.20. How many quarters do I have? (The question makes this problem nonessential for this indicator)

because it involves solving the system rather than setting up the system from the given information.)

- I have eight coins consisting of quarters, dimes and nickels. The sum of money I have is \$1. I have twice as many dimes as nickels. Write a system of equations that could be solved to determine how many of each coin I have.

- **Misconceptions/Common Errors**

None Noted

- **Technology**

Use technology where appropriate.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to analyze given information to write an appropriate system of equations to model a situation. Assessment should focus on writing a system of linear equations that models the problem situation.

- **Assessment Item Examples**

- There are 24 students enrolled in Algebra class. There are 3 fewer girls than twice the number of boys. Which system of equations can be used to find g , the number of girls who were present and b , the number of boys in the class?
 - A. $g + b = 24$
 $g = 2b - 3$
 - B. $g + b = 24$
 $b = 2g - 3$
 - C. $g + b = 24$
 $g = 2b + 3$
 - D. $g + b = 24$
 $b = 2g + 3$
- John and his sister Kim have separate savings accounts for college. John has a balance of \$2000 in his account and plans to deposit \$50 per month into the account. Kim has a balance of \$1600 in her account and plans to deposit \$100 per month. Write a system of equations to model when the two will have the same amount in their college accounts. A represents the balance in the account and t is time in months.
 - A. $A = 50t + 2000$
 $A = 100t + 1600$
 - B. $A = 2000t + 50$

$$A = 1600t + 100$$

C. $A = 50t + 100$
 $A = 2000t + 160$

D. $A = 100t + 50$
 $A = 1600t + 2000$

IV. Resources

Lesson # 2
Topic: Solving a linear system graphically
Standard (s): EA – 4.9

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In 8th grade, students translate among verbal, graphic, tabular and algebraic representations of linear functions (8-3.1) which include generating a table of values from a given equation and graphing those values on the coordinate plane.
 - In Elementary Algebra, students carry out a procedure to solve systems of linear equations graphically.
 - In Intermediate Algebra, students carry out a procedure to solve systems of linear inequalities algebraically (IA 2.1) and graphically (IA-2.2). Students transfer their knowledge of systems to carry out a procedure to solve systems involving one linear and one quadratic function (IA-2.11).
- **Taxonomy**
 - 3.1-C
 - Cognitive Process Dimension: Apply
 - Knowledge Dimension: Procedural Knowledge
- **Key Concepts**
 - System of equation
 - Solution to a linear system

II. Teaching the Lesson

In this lesson, students solve linear systems graphically. Beginning with a problem that requires students to solve a linear system in order to solve the problem provides a context in which the process can be used. In addition, students interpret the meaning of the solution.

- **Essential Learning and Understanding**

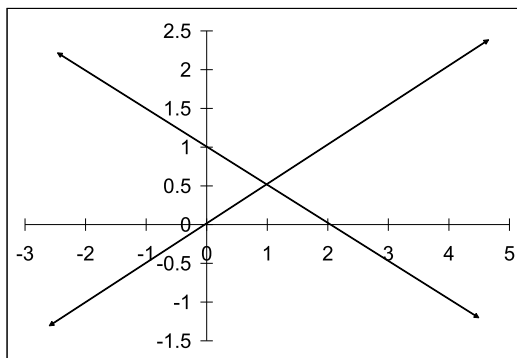
It is essential for students to do the following for the attainment of this indicator:

- Graph a linear equation
- Determine the point of intersection of the graph of the two linear equations
- Recognize and understand when a system has exactly one solution, infinitely many solutions or no solution
- Check the solution to a linear system

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Solve by graphing.
$$\begin{array}{rcl} 3x + 2y & = & 4 \\ -x + 3y & = & -5 \end{array}$$
- Does the system have one solution, infinitely many solutions or no solution?
$$\begin{array}{rcl} -a - 2b & = & 5 \\ 2a + 4b & = & 10 \end{array}$$
- Which system of linear equations is graphed below?
$$\begin{array}{llll} a. & -4x + 3y = 2 & b. & -x + 2y = 2 \\ & -2x + y = 1 & c. & 2x + y = 1 \\ & & & d. & x + 2y = 2 \\ & & & & x - 2y = 0 \end{array}$$



- Which ordered pair is a solution of the following system of linear equations?
$$m + n = 3$$
$$2m + n = 6$$

a. (1,2) b. (0,3) c. (2,1) d. (3,0)

- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Graph a system of linear equations containing more two linear equations
- Graph a system of linear equations containing more than two variables

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

Solve the system.

$$3x - 2y = 11$$

$$-x + 6y = 7$$

$$x + y = 7$$

Solve the system.

$$x - 2y + 3z = 7$$

$$2x + y + z = 4$$

$$-6x + 4y - 4z = -20$$

- **Misconceptions/Common Errors**

- Students may verify the solution by substituting the resulting ordered pair into an equation other than the original equation. If an error occurs in solving the system, substituting the ordered pair in the incorrect form will yield incorrect result. The solution should be verified using the original equations.

- **Technology**

- Students may check their solutions by direct substitution into the system.

- Students may check their solutions by using the intersect feature on a graphing utility in order to determine where the graphs of the two linear functions intersect.
- The table of values can also be used to verify for which x value are the y values of both equations equal. The table will need to be set in order to display the appropriate values.
- One of the difficulties students may have when using a graphing utility to determine the point of intersection is setting an appropriate viewing window that clearly displays the point of intersection. Students need sufficient practice performing this skill.

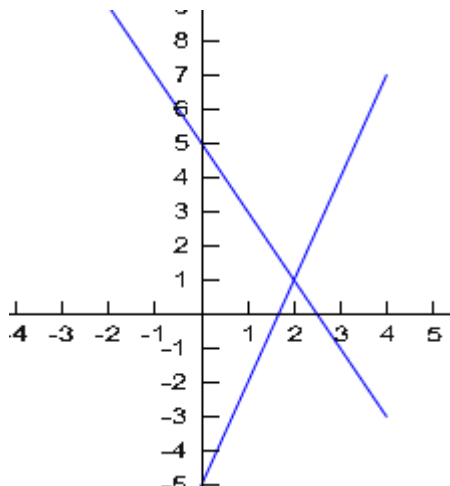
III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is for the student to carry out a procedure to solve systems of two linear equations graphically. Therefore, the primary focus of the assessment should be for students to carry out such procedures.*

- **Assessment Item Examples**

- Which system of linear equations is graphed below?
 - A. $3x + y = 9$
 $6x + 2y = 6$
 - B. $2x + y = 1$
 $6x + 3y = 3$
 - C. $-2x + y = 1$
 $3x - 2y = 5$
 - D. $y - 3x = -5$
 $2x + y = 5$

Elementary Algebra Instructional Planning Guide
Module 7 Systems of Linear Equations



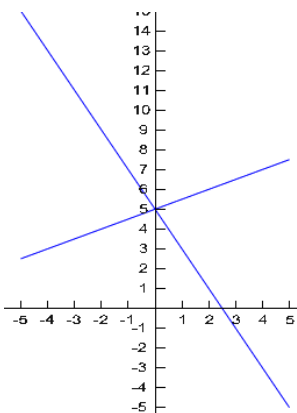
- What ordered pair is the solution to the system graphed above?
A. (2,1) B. (1,2) C. (0, 5) D. (0,-5)

- Solve by graphing.

$$2x + y = 1$$

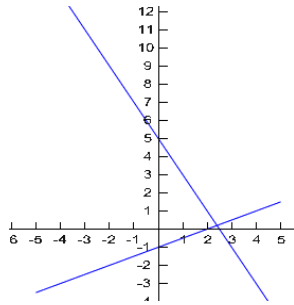
$$6x + 3y = 3$$

A.

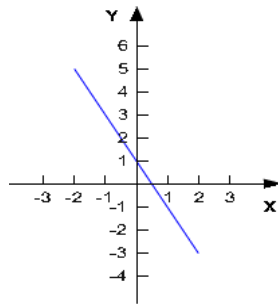


B.

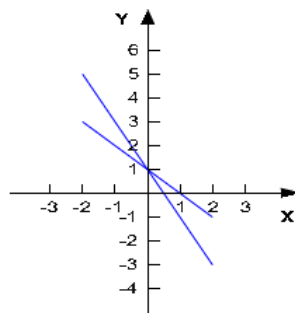
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C.



D.



IV. Resources

Lesson # 3
Topic: Solving systems of linear equations algebraically
Standard (s): EA – 4.10

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
 - In 8th grade, students translate among verbal, graphic, tabular and algebraic representations of linear functions (8-3.1) which

include generating a table of values from a given equation and graphing those values on the coordinate plane.

- In elementary algebra, students carry out a procedure to solve systems of linear equations algebraically.
- In Intermediate Algebra, students carry out a procedure to solve systems of linear inequalities algebraically (IA 2.1) and graphically (IA-2.2). Students transfer their knowledge of systems in order to carry out a procedure to solve systems involving one linear and one quadratic function (IA-2.11).

- **Taxonomy**

3.1-C

Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

- **Key Concepts**

System of equation

Substitution Method

Elimination Method (Linear combination)

Consistent

Dependent

Independent

Inconsistent

II. Teaching the Lesson

In this lesson, students solve linear systems algebraically. The contextual problem in the introduction can also be used to demonstrate the algebraic processes of solving linear systems.

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Multiply a linear equation by a constant to create an equivalent linear equation
- Understand and apply the principle of the additive inverse
- Solve one step linear equations
- Apply the substitution or elimination method to solve a system
- Recognize and understand when a system has exactly one solution, infinitely many solutions or no solution
- Check solutions to the system using an appropriate method

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

Solve. $x = y - 1$
 $2x + y = -2$

Solve. $a + 4b = 23$
 $-a + b = 2$

Solve. $p - 3q = 30$
 $3q + p = 12$

Solve. $s = t + 4$
 $2t + s = 19$

Verify whether the ordered pair, $(-4, 23)$, is a solution to the system.

$$2x + y = 15$$

$$-x + y = 12$$

Does the system have one solution, no solution or infinitely many solutions?

$$3x + 4y = 8$$

$$-3x - 4y = 10$$

- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

- Solve a linear system containing more than two linear equations.
- Solve a linear system containing more than two variables

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

Solve the system

$$2a + b = 10$$

$$5a - b = 18$$

$$-4a + b = -14$$

Solve the system

$$x - 2y + 3z = 7$$

$$2x + y + z = 4$$

$$-6x + 4y - 4z = -20$$

- **Misconceptions/Common Errors**

- Students may eliminate a variable because their coefficients have opposite signs without making sure that the two expressions are additive inverses.
- Students may verify the solution by substituting the resulting ordered pair into equation other than the original equation. If an error occurs in solving the system, substituting the ordered pair in the incorrect form will yield incorrect result. The solution should be verified using the original equations.

- **Technology**

Students can check their solutions by direct substitution or by using the intersect feature on a graphing calculator.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is for the student to carry out a procedure; therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples**

Solve:
$$\begin{aligned} x + y &= -2 \\ 2x - 3y &= 9 \end{aligned}$$

- A. (3,1) B. (1,3) C. (-3,1) D. (1,-3)

Solve:
$$\begin{aligned} 3x + y &= 11 \\ x - 2y &= 6 \end{aligned}$$

- A. (4,-1) B. (0,4) C. (-1,4) D. (4,0)

Determine the number of solutions for the system of equations below.

$$\begin{aligned} 2x + y &= 1 \\ 6x + 3y &= 3 \end{aligned}$$

- A. One Solution B. Two Solutions
B. C. No solution D. Multiple solutions

IV. Resources

Lesson # 4
Topic: Determining whether two lines are parallel, perpendicular or neither
Standard (s): EA – 5.8

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**

- In 7th grade students analyze the congruent and supplementary relationships—specifically, alternate interior, alternate exterior, corresponding, and adjacent—of the angles formed by parallel lines and a transversal (7-4.5).
- In Elementary Algebra students analyze the equations of two lines to determine whether the lines are perpendicular or parallel.
- In Geometry students apply properties of parallel lines, intersecting lines, and parallel lines cut by a transversal to solve problems (G-2.2). Also, students carry out a procedure to create geometric constructions (including the midpoint of a line segment, the angle bisector, the perpendicular bisector of a line segment, the line through a given point that is parallel to a given line, and the line through a given point that is perpendicular to a given line) (G-2.5).

- **Taxonomy**

4.1-B

Cognitive Process Dimension: Analyze

Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**

Slope

Parallel lines

Perpendicular lines

Reciprocal

II. Teaching the Lesson

In this lesson, students determine if two lines are parallel, perpendicular or neither. This concept can be related back to solving linear systems by using this process to determine if a linear system does (perpendicular lines) or does not have (parallel lines) have a solution. If the slopes are equal then students can examine the y-intercept to determine if the two equations are equivalent (infinitely many solutions).

- **Essential Learning and Understanding**

It is essential for students to do the following for the attainment of this indicator:

- Know the relationship between the slopes of two parallel lines.
- Know the relationship between the slopes of two perpendicular lines.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

- Are the two lines $y = 3x - 5$ and $y = 3x + 5$ parallel?
- Are the two lines $y = 3x - 5$ and $y = -3x + 5$ perpendicular?
- Classify the two lines as parallel, perpendicular or neither:
 $y = 3x - 5$ and $y = -3x - 5$
- Classify the two lines as parallel, perpendicular or neither:
 $y = 3x - 5$ and $y = 3x + 5$
- Classify the two lines as parallel, perpendicular or neither:
 $y = 3x - 5$ and $y = -\frac{1}{3}x - 5$
- Which of the lines whose equations are given below is perpendicular to $y = 3x - 2$?
a) $y = 3x + 5$ b) $y = -3x + 5$
c) $y = \frac{1}{3}x - 5$ d) $y = -\frac{1}{3}x - 5$
e) none of these

- **Non-Essential Learning and Understand**

It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

Find the equation of a parallel or perpendicular line to a given line.

- **Examples of Non-Essential Tasks**

The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- Write the equation of a line that is parallel to $y = 3x - 2$.
- Write the equation of a line that is perpendicular to $y = 3x - 2$

- **Misconceptions/Common Errors**

- Students often choose negative or the reciprocal slope instead of the negative reciprocal slope when identifying perpendicular lines.

- Students often mistake slope for y-intercept if the equation is written in $y = b + mx$ form.

- **Technology**

Use technology where appropriate.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to analyze given equations to determine if two lines are parallel or perpendicular.

- **Assessment Item Examples**

What is the relationship between the lines $y = 2x - 5$ and $y = 2x + 5$?

- A. The lines are parallel.
- B. The lines are perpendicular.
- C. The lines are the same line.
- D. There is no relationship.

What is the relationship between the lines $7x + 5y = 16$ and $14x + 10y = 32$?

- A. The lines are parallel.
- B. The lines are perpendicular.
- C. The lines are the same line.
- D. There is no relationship.

What is the relationship between the lines $y = 5x + 2$ and $y = -\frac{1}{5}x + 2$?

- A. The lines are parallel.
- B. The lines are perpendicular.
- C. The lines are the same line.
- D. There is no relationship.

Which of the following equations is perpendicular to $y = 6x + 9$?

- A. $y = 6x + 18$
- B. $y = \frac{1}{6}x + 9$
- C. $y = -6x + 18$
- D. $y = -\frac{1}{6}x + 9$

IV. Resources